Order of Magnitude

The order of magnitude of a number is the number expressed to the nearest ~~whole number~~ integer power of 10.

Pop of Earth: 6.9 billion 6.9×10^9

between 10^9 and 10^{10}

order of magnitude is 10^{10}

Gravitational field strength $9.8\,N\,kg^{-1}$

b/w 10^0 and 10^1

order of magnitude is 10^1 or 10

Mass of an electron: 9.1×10^{-31} kg

b/w 10^{-31} and 10^{-30}

order of magnitude is 10^{-30}

Try these:

1. Atmospheric Pressure at sea level 1.01×10^5 Pa — 10^5 or $\cancel{10^6}$
2. Average height of a adult male 1.784 m — 10^0 or 10^1
3. Box of apples has a mass of 16 kg — 10^1 or 10^2
4. Cheese 0.075 kg — 7.5×10^{-2} — 10^{-2} or 10^{-1} — 0.01 — 0.1

Rounding with orders of magnitude:

cut off 3.16

Try these:

4.13×10^3 — $\cancel{10^3}$ 10^4 ✓

3.09×10^{-6} — 10^{-6} $\cancel{10^{-5}}$

4245 — 10^3 10^4 — $3.3 > 3.16$

0.00033 — 10^{-4} 10^{-3} — 3.3×10^{-4}

Example:
Which of the following is the same as 3.8×10^{-3} to the nearest order of magnitude? $\cancel{10^{-3}}$ or 10^{-2}

a) 10^{-3} 3.0×10^{-3}

b) 10^{-1} 4.0×10^{-2}

c) 10^{-2}

d) 10^{-3}

$3.8 > 3.16$
round up the order of magnitude.

Ratios are expressed as a difference in orders of magnitude:

Compare

$$\frac{\text{diameter of H atom}}{\text{diameter of H nucleus}} = \frac{10^{-10}}{10^{-15}} = 10^{5}$$

the diameter of H atom is 10^5 orders of magnitude larger than the diameter of the nucleus

Example:

The diameter of a proton is about 10^{-15}m and the diameter of a hydrogen atom is about 10^{-10}m.

How many orders of magnitude is the volume of a hydrogen atom greater than the volume of its nucleus?

just a proton for Hydrogen

$V = \frac{4}{3}\pi r^3$

$$\frac{\text{volume of H atom}}{\text{volume of nucleus}} = \frac{\cancel{\frac{4}{3}\pi}\left(\frac{d_{atom}}{2}\right)^3}{\cancel{\frac{4}{3}\pi}\left(\frac{d_{nuc}}{2}\right)^3}$$

$$= \frac{\frac{d_{atom}^{3}}{\cancel{2^3}}}{\frac{d_{nuc}^{3}}{\cancel{2^3}}}$$

$$= \frac{d_{atom}^{3}}{d_{nuc}^{3}}$$

$$= \frac{(10^{-10})^3}{(10^{-15})^3}$$

$$= \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$$= (10^5)^3$$

$$= 10^{15}$$

The volume of the hydrogen atom is 10^{15} times bigger than its nucleus

(a 15 orders of magnitude bigger)

Example

How many orders of magnitude is the length of a metre stick longer than the width of a pencil?

$$\frac{1\text{m}}{0.01\text{m}} = 100 = 10^2$$

$\to \frac{10^0}{10^{-2}} = 10^2$

2 orders of magnitude longer

Estimate the following to the nearest order of magnitude

10^5 $\cancel{10^4}$ $\cancel{10^{-4}}$ 10^6 $\frac{10^5}{10^6} = 10^{-1}$

(1) 47816 x (4293 x 10^{-4}) / 403000

4293 EE -4 $= 0.0509\ldots$

(2) $\sqrt{\dfrac{2\pi}{4.6\times10^{-5}}}$ 10^1 10^{-4}

$\sqrt{\dfrac{10^1}{10^{-4}}} = \sqrt{10^5} = \left(10^5\right)^{1/2} = 10^{5/2}$

does this look familiar → 316

3.16×10^2

$10^{2.5}$

10^2 or 10^3

Estimate to 1 or 2 significant digits or nearest order of magnitude the size of everyday objects.

- estimate familiar lengths, masses, weights + times
- estimate based on a scale diagram
- rough estimates for calculation
- trace any error between the estimated + calculated

Examples

10^0 10^1 10^1

3 cm x 20 cm x 30 cm

- dimensions of your physics book in cm (1 sf)
- mass of an apple in kg (1 sf) 2×10^{-1} kg 10^{-1}
- period of a heart beat in s (to 1 sf) 1 s 10^0
- quantity of milk you drink in a year in cm^3 (to 1sf) 5×10^5 cm^3 10^6

Estimate the following to the nearest order of magnitude

$10^5 \times \cancel{10^4} \times \cancel{10^{-4}} \div 10^6$

(1) 47816 x (4293 x 10^{-4}) / 403000 $\quad 10^{-1} \quad$ (0.050936.....)

4293 E-4

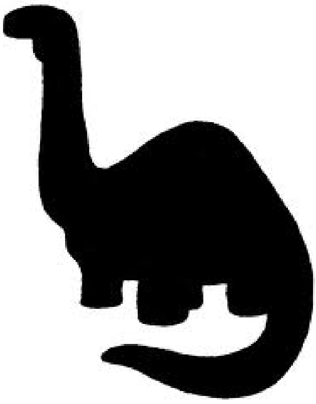
10^{-2} and 10^{-1}

(2) $\sqrt{\dfrac{2\pi}{4.6 \times 10^{-5}}}$ $\quad \dfrac{10^1}{10^{-4}}$

$\sqrt{\dfrac{10^1}{10^{-4}}} = \sqrt{10^5}$

$= 10^{5/2} \quad 10^2$ or 10^3

316 ↑ middle



10^2

Estimate the mass of the dinosaur in kg to the nearest order of magnitude. State any assumptions that you have made.

10^4 Coleman MH CC

10^4 Sam, Ross, Chelsea

10^5 Lydia + Lucy

10^3 Martin

$\cancel{10^1}$

$\cancel{10^2}$

10^3

10^4

10^5

$\cancel{10^6}$

$\cancel{10^{10}}$

Scientific Notation + Metric Conversions

- Express 1.2×10^{-9} s in units of ns => 1.2 ns

  n $\quad 10^{-9}$

- Express 47 GJ in J in "standard form" (scientific notation)

  $\times 10^{9}$

  $$47 \times 10^{9}\,J = 4.7 \times 10^{10}\,J$$

- Express 4.3×10^{-7} m (the wavelength of violet light) in nm.

  $\to 430 \times 10^{-9}\,m = 430\,nm$

  n

  10^{2}

- Express 1.0×10^{5} Pa (atmospheric pressure) in hPa

  h

  $\to 1000 \times 10^{2}\,Pa = 1000\,hPa$

  $1.0 \times 10^{3}\,hPa$

  10^{-6} ← make 3 dec places smaller 10^{3}

- Express 1.5 μg in kg

  $$1.5 \times 10^{-9}\,kg$$

  10^{-9} $\underbrace{\quad}_{}$ 3 dec 10^{-6}

- 25 nm to μm

  0.025 μm

  2.5×10^{-2} μm

Convert:

20 cm^2 to m^2

1 cm = 10^{-2} m

" 10^{-2} m per cm "

$$20\cancel{cm^2}\left(10^{-2}\,m\,cm^{-1}\right)^2 = 20\times10^{-4}\,m^2$$

m^2 ~~cm^{-2}~~

$2.0\times10^{-3}\,m^2$

$$20\,cm^2\left(\frac{1m}{100cm}\right)^2 = 0.0020\,m^2$$

Convert:

~~3600 s^{-1}~~

3600^{-1} s-1

$$60\cancel{km}\,\cancel{h^{-1}}\left(1000\,m\,\cancel{km^{-1}}\right)\left(1\cancel{h}\,(3600\,s)^{-1}\right)$$

$$60\,km\,h^{-1}\left(\frac{1000\,m}{km}\right)\left(\frac{1h}{3600s}\right)$$

$\boxed{17\,ms^{-1}}$

Example

How many joules of energy are there in one kilowatt · hour

kW (power) h (time)

$$\text{Power} = \frac{\text{work}}{\text{time}}$$

$1\,W = 1\,J\,s^{-1}$

$$\text{Work (energy)} = \text{power} \times \text{time}$$

$$= 1\,kW \cdot 1\,h$$

$$= 1000\,J\,\cancel{s^{-1}}\;3600\,\cancel{s}$$

$$= 3.6 \times 10^{6}\,J \quad (3.6\,MJ)$$

Conversion factor: $\boxed{1\,kW\,h = 3.6 \times 10^{6}\,J}$

Examples

Convert an energy of 120 kW h to J:

$$120\,\cancel{kW}\,\cancel{h}\left(3.6 \times 10^{6}\,J\;\cancel{kW^{-1}}\,\cancel{h^{-1}}\right)$$

$$4.3 \times 10^{8}\,J$$

How many kW h of energy are produced if the work done is $7.2 \times 10^{8}\,J$?

$$7.2 \times 10^{8}\,J\left(1\,kW\,h\,(3.6 \times 10^{6}\,J)^{-1}\right)$$

$$200\,kW\,h$$

Another common conversion factor is for the electronvolt:

$$1\,eV = 1.6 \times 10^{-19}\,J$$

(More about this in TOPIC 5)